

3.8 Fundamental Identities

Name _____ Date _____ Period _____

Fill in the blank with the appropriate term (even or odd).

1. If $f(x) = f(-x)$, then f is an _____ function.
2. If $f(-x) = -f(x)$, then f is an _____ function.
3. Sine, cosecant, tangent, and cotangent are _____ functions.
4. Cosine and secant are _____ functions.

Write each expression in terms of sines and/or cosines, and then simplify.

5. $\frac{\sec x}{\tan x}$

6. $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x}$

7. $(1 - \sin \alpha)(1 + \sin \alpha)$

8. $(\cos \beta \tan \beta + 1)(\sin \beta - 1)$

9. $\frac{1 + \cos \alpha \tan \alpha \csc \alpha}{\csc \alpha}$

10. $\frac{(\cos \alpha \tan \alpha + 1)(\sin \alpha - 1)}{\cos^2 \alpha}$

In each exercise, use identities (remember: $\sin \alpha = \frac{y}{r}$, $\cos \alpha = \frac{x}{r}$, $\tan \alpha = \frac{y}{x}$, etc.) to find the exact values at α for the remaining five trigonometric functions.

11. $\tan \alpha = \frac{1}{2}$ and $0 < \alpha < \frac{\pi}{2}$

12. $\sin \alpha = \frac{3}{4}$ and $\frac{\pi}{2} < \alpha < \pi$

13. $\cos \alpha = \frac{-\sqrt{3}}{5}$ and α is in quadrant III

14. $\cot \alpha = \frac{-1}{3}$ and $-\frac{\pi}{2} < \alpha < 0$

Simplify each expression using the basic identities and even and odd identities.

15. $\sin(-x)\cot(-x)$

16. $\sec(-x) - \sec(x)$

17. $(1 + \sin(\alpha))(1 + \sin(-\alpha))$

18. $\sin(-\beta)\cos(-\beta)\csc(\beta)$

Use identities to simplify each expression.

19. $1 - \frac{1}{\cos^2 x}$

20. $\frac{\sin^2 \alpha - \cos^2 \alpha}{1 - 2\cos^2 \alpha}$

21. $\frac{1}{\sin^3 x} - \frac{\cot^2 x}{\sin x}$

22. $\sin^4 x - \cos^4 x$

Review Exercises

23. Find the smallest positive angle that is coterminal with -35° .

24. Convert each degree measure to radian measure.

a) 225°

b) -210°

c) 270°

25. Find the exact area of a sector of the circle with radius 6 feet and central angle 15° .

26. Evaluate without using a calculator. Some of these expressions are undefined.

a) $\cos\left(\frac{-3\pi}{2}\right)$

b) $\sin\left(\frac{9\pi}{4}\right)$

c) $\tan\left(\frac{-3\pi}{2}\right)$

d) $\tan(3\pi)$

e) $\sec\left(\frac{13\pi}{6}\right)$

f) $\csc\left(\frac{5\pi}{2}\right)$

g) $\cot\left(\frac{-5\pi}{3}\right)$

h) $\sin\left(\frac{3\pi}{2}\right)$

i) $\cos\left(\frac{7\pi}{6}\right)$

27. Find the amplitude, phase shift, frequency, period, and range of the given function.

$$f(x) = -3\sin\left[5\left(x - \frac{\pi}{2}\right)\right] + 7$$